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# Optimal Privatization Policy with Asymmetry among Private Firms<sup>\*</sup>

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## Abstract

We revisit the relationship between the optimal privatization policy and market competition indexes such as the Hirschman–Herfindahl index, which is affected by the number of firms and asymmetry of size among these firms: the larger the number of firms (the less asymmetry among firms), the lower the market concentration index. The literature on mixed oligopolies suggests that the optimal degree of privatization is increasing with the number of private firms (and, thus, decreasing with the market competition index), assuming that all private firms are homogeneous. We investigate how the asymmetry among private firms affects the optimal degree of privatization. We propose the simplest and natural model formulation for discussing asymmetry among private firms. We find that the optimal degree of privatization is either nonmonotone or monopolistically increasing (and, thus, never monopolistically decreasing) in the asymmetry among private firms.

**JEL classification numbers:** H44, L33, L44

**Key words:** market concentration index, asymmetry of private firms, mixed oligopolies

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# 1 Introduction

The Hirschman–Herfindahl index (HHI) is the most influential indicator of the degree of market concentration and the amount of competition in an industry. It is widely applied in public policies such as competition law, antitrust, and regulations.<sup>1</sup> It is defined as the sum of the squares of the market shares of the firms within the industry and is affected by the number of the firms and the asymmetry among the sizes of the firms; the larger the number of firms (i.e., the less asymmetry among firms), the lower the market concentration index.

In the literature on mixed oligopolies<sup>2</sup>, many papers have already investigated the relationship between the optimal privatization policy and the market concentration index by examining how the number of the private firms affects the privatization policy.<sup>3</sup> De Fraja and Delbono (1989) formulated a model of mixed oligopolies in which a public enterprise competes against  $n$  private firms in a homogeneous product market. They assumed that both public and private firms have an identical cost function and showed that the full privatization more likely improves welfare when  $n$  is larger. Matsumura and Shimizu (2010) showed that this result holds even when multiple public firms exist and the cost difference between public and private firms is allowed, regardless of whether the products are homogeneous or differentiated. Lin and Matsumura (2012) adopted a partial privatization approach formulated by Matsumura (1998) and showed that the optimal degree of privatization is increasing with the number of private firms. Matsumura and Okamura (2015) showed that this is true even when private firms maximize relative profit rather than absolute profits.<sup>4</sup>

All the papers mentioned above assumed that private firms are symmetric (homogeneous).

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<sup>1</sup>See Viscusi *et al.* (2005).

<sup>2</sup>For important examples of mixed oligopolies and recent development of the analysis of mixed oligopolies, see Ishida and Matsushima (2009), Gil-Molto *et al.* (2011), Bose *et al.* (2014), and Matsumura and Tomaru (2015).

<sup>3</sup>Throughout this study, we define HHI as the sum of the squares of the market shares of the private firms among private firms. The output of the public firm is directly affected by the privatization policy, and thus, we exclude it to discuss the relationship between market index and the optimal privatization policy.

<sup>4</sup>The relative profit maximization approach enables us to treat various competition structures, from collusive to perfectly competitive cases, by the single quantity competition model. Thus, this result implies that the optimal degree of privatization is increasing with the number of private firms under various competition structures. For the discussion of relative profit maximization, see Matsumura and Matsushima (2012) and Matsumura *et al.* (2013).

Therefore, these results suggest that the lesser the market is concentrated, the more the government should privatize the public firms. However, as mentioned above, the market concentration index also depends on the heterogeneity among firms. Thus, we should more carefully investigate the relationship between the market concentration index and the optimal privatization policy when the heterogeneity among the firms is non-negligible. For example, Japanese financial markets are a typical example of mixed oligopolies in which public financial institutions such as Development Bank of Japan, Japan Finance Corporation, Postal Bank, and Kampo compete against private banks and life-insurance companies, and there is huge heterogeneity among private banks and life-insurance companies in some markets. The Vietnamese economy is another example. Many state-owned firms in Vietnam compete against private firms across a wide variety of industries and the size of private firms are far from homogeneous (Huang and Yang, 2016).

In this study, we formulate the simplest model to discuss how heterogeneity of private enterprises affects the optimal privatization policy. We investigate a triopoly model in which one public enterprise competes against two private enterprises. We find that the optimal degree of privatization is either increasing with the degree of asymmetry between firms or nonmonotone and decreasing (increasing) when the asymmetry is small (large).

This result has an important implication because in contrast with the results in the literature on mixed oligopolies, here, it is ambiguous whether the government should privatize the public firm more when the market concentration index such as HHI is smaller. If the market concentration index is larger because of the smaller number of private firms, the government should privatize less. However, if it is larger because of the larger asymmetry among private firms, the government should privatize more. Therefore, when we consider the optimal privatization policy, we should pay more attention to the reason for the market index being large.

As mentioned above, the Japanese financial markets are typical mixed oligopolies. Postal Bank, a major public financial institution, competes with many small regional private banks with relatively homogeneous size. On the contrary, Development Bank of Japan, another major

public financial institution, competes with a smaller number of mega banks, smaller-sized regional banks, and further smaller private funds, with significant heterogeneity of the size, in the corporate loan markets and founding for corporate revitalization. Our result suggests that the optimal degree of privatization is higher in Postal Bank than in Japan Development Bank. This is consistent with the following current privatization policies in Japan. The government has already partially privatized Postal Bank and plans to reduce its ownership share in it, whereas the government postponed the privatization of Development Bank of Japan.

## 2 The Model

We formulate a mixed triopoly model. Firm 0 is a state-owned public firm. Firms 1 and 2 are private firms. These firms produce homogeneous products. The market demand function is given by

$$p(Q) = a - Q,$$

where  $p$  denotes the price,  $a$  is positive constant, and  $Q$  is the total output.

Regarding cost functions, we consider the following situation.<sup>5</sup> Each factory has the following cost structure. It takes  $F$  as the set-up cost and  $(k/2)q^2$  as the variable cost, where  $q$  is the output produced at this factory and  $k$  is a positive constant. If a firm  $i$  holds  $m_i$  factories, it allocates the same production level among factories, and thus, its cost is

$$c_i(q_i) = \frac{k}{2m_i}q_i^2 + m_iF.$$

We assume that  $m_0 = 1$  and  $m_1 + m_2 = m$ . Without loss of generality, we assume that  $m_1 \geq m/2$  or equivalently,  $m_1 \geq m_2$ . Therefore, a larger  $m_1$  implies more asymmetry between the private firms.

Each private firm's objective is its respective profit, which is given by

$$\pi_i = p(Q)q_i - c_i(q_i).$$

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<sup>5</sup>Linear demand and quadratic cost functions are very popular in the literature on mixed oligopolies. See De Fraja and Delbono (1989). See also Matsumura and Shimizu (2010) and the works cited therein.

Following the standard approach in the literature formulated by Matsumura (1998), we assume that the public firm's objective function is a convex combination of social surplus and its respective profit. This is denoted as

$$\Omega = \alpha\pi_0 + (1 - \alpha)W$$

where  $W$  is the social surplus, given by

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^2 \pi_i = \int_0^Q p(q)dq - \sum_{i=0}^2 c_i(q_i),$$

and  $\alpha \in [0, 1]$  represents the degree of privatization. In the case of full nationalization (*i.e.*,  $\alpha = 0$ ), firm 0 maximizes social welfare. In the case of full privatization (*i.e.*,  $\alpha = 1$ ), firm 0 maximizes its own profit.

The game runs as follows. In the first stage, the government chooses the degree of privatization  $\alpha$  to maximize the social surplus. In the second stage, each firm simultaneously chooses its output to maximize its objective. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

### 3 Results

First, we solve the second-stage game, given  $\alpha$ . The first-order conditions of public and private firms are, respectively,

$$\frac{\partial \Omega}{\partial q_0} = a - (1 + \alpha + k)q_0 - q_1 - q_2 = 0, \quad (1)$$

$$\frac{\partial \pi_1}{\partial q_1} = a - (2 + \frac{k}{m_1})q_1 - q_0 - q_2 = 0, \quad (2)$$

$$\frac{\partial \pi_1}{\partial q_1} = a - (2 + \frac{k}{m - m_1})q_2 - q_0 - q_1 = 0. \quad (3)$$

The second-order conditions are satisfied. From (1)–(3), we obtain the following reaction functions of public and private firms, respectively:

$$\begin{aligned} R_0(q_1, q_2) &= \frac{a - q_1 - q_2}{1 + \alpha + k}, \\ R_1(q_0, q_2) &= \frac{m_1(a - q_0 - q_2)}{2m_1 + k}, \\ R_2(q_0, q_1) &= \frac{(m - m_1)(a - q_0 - q_1)}{2(m - m_1) + k}. \end{aligned}$$

These reaction functions lead to the following equilibrium quantities of public and private firms, respectively:

$$\begin{aligned} q_0^* &= \frac{a(m_1 + k)(k + m - m_1)}{(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}}, \\ q_1^* &= \frac{am_1(k + \alpha)(k + m - m_1)}{(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}}, \\ q_2^* &= \frac{a(m_1 + k)(k + \alpha)(m - m_1)}{(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}}. \end{aligned}$$

The resulting equilibrium total output, price, and welfare are, respectively,

$$\begin{aligned} Q^* &= \frac{a\{m_1(2k + 2\alpha + 1)(m - m_1) + (1 + \alpha)km + k^2(m + 1)\}}{(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}}, \\ p^* &= \frac{a(k + \alpha)(k + m_1)(k + m - m_1)}{(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}}, \\ W^* &= \frac{a^2 X_1}{2[(3k + 3\alpha + 1)(m - m_1)m_1 + k\{(2k + 2\alpha + 1)m + (k + \alpha + 1)k\}]^2} - (m + 1)F, \end{aligned}$$

where  $X_1 \equiv \{8(k + \alpha) + 5k + 6\alpha + 1\}(m - m_1)^2 m_1^2 + k\{9(k + \alpha)^2 + 8k + 10\alpha + 2\}(m - m_1)mm_1 + 2k^2(3k + 4\alpha + 1)(m - m_1)m_1 + k^2\{3(k + \alpha)^2 + 3k + 4\alpha + 1\}m^2 + k^3\{(\alpha + k)^2 + 4k + 6\alpha + 2\}m + k^4(k + 2\alpha + 1)$ .

We obtain the following result.

**Proposition 1** (i)  $q_0^*$  is increasing in  $m_1$ . (ii)  $q_1^*$  is increasing in  $m_1$ . (iii)  $q_2^*$  is decreasing in  $m_1$ . (iv)  $q_1^* + q_2^*$  is decreasing in  $m_1$ . (v)  $Q^*$  is decreasing in  $m_1$ .

**Proof**

Taking the first-order derivative of equilibrium quantities of public and private firms and equi-

librium total output with respect to  $m_1$ , we obtain

$$\begin{aligned}
\frac{\partial q_0^*}{\partial m_1} &= \frac{ak(k+\alpha)(m+2k)(2m_1-m)}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} > 0, \\
\frac{\partial q_1^*}{\partial m_1} &= \frac{ak(k+\alpha)X_2}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} > 0, \\
\frac{\partial q_2^*}{\partial m_1} &= -\frac{ak(k+\alpha)X_3}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} < 0, \\
\frac{\partial q_1^*}{\partial m_1} + \frac{\partial q_2^*}{\partial m_1} &= -\frac{ak(k+\alpha)(k+\alpha+1)(m+2k)(2m_1-m)}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} < 0, \\
\frac{\partial Q^*}{\partial m_1} &= -\frac{ak(k+\alpha)^2(m+2k)(2m_1-m)}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} < 0,
\end{aligned}$$

where

$$\begin{aligned}
X_2 &\equiv \{2(\alpha+k)+1\}(m-m_1)^2 + 2k(\alpha+k+1)(m-m_1) + (\alpha+k)m_1^2 + k(1+k)m + k^2(1+\alpha) + k^3, \\
X_3 &\equiv (k+\alpha)\{(m-m_1)^2 + m_1^2\} + m_1^2 + k\{\alpha(m+m_1) + 2m_1\} + k^2(1+\alpha+m+2m_1) + k^3.
\end{aligned}$$

These results imply Proposition 1. ■

An increase in  $m_1$  reduces (raises) firm 1's (firm 2's) marginal cost, and thus, the output of firm 1 (firm 2) is increasing (decreasing) in  $m_1$  (Proposition 1(ii) and (iii)). Proposition 1(iv) states that the total output of the private firms is decreasing in  $m_1$ , and this result (a higher rate of market concentration among private firms makes the market less competitive) is very natural. Because firm 0's reaction curve has a negative slope (strategic substitute), firm 0 expands its output as  $m_1$  increases, responding to the reduction of the private firms' outputs (Proposition 1(i)). The direct effect of the reduction of the private firms' outputs dominates the indirect effect of the increase of the public firm's output, and thus, the total output decreases as  $m_1$  increases (Proposition 1(v)). Because the price is increasing in  $m_1$ , we believe that  $m_1$  is an appropriate parameter reflecting the intensity of market competition.

We now present a relationship between price-cost margins and  $m_1$ .

**Proposition 2** (i)  $p^* - c'_0(q_0^*)$  is nondecreasing in  $m_1$  and increasing in  $m_1$  for  $\alpha > 0$ . (ii)  $p^* - c'_1(q_1^*)$  is increasing in  $m_1$ . (iii)  $p^* - c'_2(q_2^*)$  is decreasing in  $m_1$ .



## Proof

Taking the difference between the equilibrium price and the marginal cost of the public and the private firms, we obtain

$$\begin{aligned} p^* - c'_0(q_0^*) &= \frac{a\alpha(k+m_1)(k+m-m_1)}{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)} \equiv f_0(m_1), \\ p^* - c'_1(q_1^*) &= \frac{a(k+\alpha)m_1(k+m-m_1)}{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)} \equiv f_1(m_1), \\ p^* - c'_2(q_2^*) &= \frac{a(k+\alpha)(k+m_1)(m-m_1)}{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)} \equiv f_2(m_1). \end{aligned}$$

Differentiating these outcomes with respect to  $m_1$  leads to the following results:

$$\frac{\partial f_0(m_1)}{\partial m_1} = \frac{a\alpha k(k+\alpha)(2k+m)(2m_1-m)}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} > 0, \quad (4)$$

$$\frac{\partial f_1(m_1)}{\partial m_1} = \frac{ak(k+\alpha)X_4}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} > 0, \quad (5)$$

$$\frac{\partial f_2(m_1)}{\partial m_1} = -\frac{ak(k+\alpha)[(k+\alpha)\{(m-m_1)^2+km\}+(k+\alpha+1)(k+m_1)^2]}{\{(3k+3\alpha+1)(m-m_1)m_1+k(2k+2\alpha+1)m-k^2(k+\alpha+1)\}^2} < 0, \quad (6)$$

where  $X_4 \equiv \{2(k+\alpha)+1\}(m-m_1)^2+(k+\alpha)(m_1^2+km)+2k(k+\alpha+1)(m-m_1)+k^2(k+\alpha+1)$ .

The strict inequality in (4) is satisfied if  $\alpha > 0$ . These imply Proposition 2. ■

The larger the private firm, the larger is the difference between the price and marginal revenue (i.e.,  $p - p'q_i$ ). Therefore, the price-cost margin of firm 1 (firm 2) is increasing (decreasing) in  $m_1$ . Because the public firm's marginal cost is independent of  $m_1$  and the price is increasing in  $m_1$ , the price-cost margin of firm 0 is increasing in  $m_1$  as long as  $\alpha > 0$ .<sup>6</sup>

Next, we discuss the government's welfare maximization problem in the first stage. From the first-order condition  $\partial W^*/\partial \alpha = 0$ , we obtain<sup>7</sup>

$$\alpha^* = \frac{2km_1^2(m-m_1)^2+2k^2(m-k)(m-m_1)m_1+k^3m^2}{m_1^2(m-m_1)^2+3k(m+2k)(m-m_1)m_1+k^2(m^2+3km+k^2)}. \quad (7)$$

We now present our main result, which shows the relationship between  $\alpha^*$  and  $m_1$ .

**Proposition 3** (i) If  $m \leq m^* \equiv (\sqrt{17}+1)k/4$ , then  $\alpha^*$  is increasing in  $m_1$  for  $(\frac{m}{2}, m]$ . (ii) If

<sup>6</sup>If  $\alpha = 0$ , welfare-maximizing public firm (firm 0) chooses its quantity so as to equalize its marginal cost to the price, and thus, price-cost margin of firm 0 is zero in equilibrium, regardless of  $m_1$ .

<sup>7</sup>The second-order condition is satisfied.

$m > m^*$ , then the relationship between  $m_1$  and  $\alpha^*$  is nonmonotone. When  $m_1$  is close to  $m/2$  ( $m$ ),  $\alpha^*$  is decreasing (increasing) in  $m_1$ .

**Proof** From (7), we obtain

$$\frac{\partial \alpha^*}{\partial m_1} = \frac{k^2(2m_1 - m)X_5}{\{(m - m_1)^2 m_1^2 + 3k(m + 2k)(m - m_1)m_1 + k^2(m^2 + 3km + k^2)\}^2}, \quad (8)$$

where

$$\begin{aligned} X_5(m_1) &\equiv 2(2m + 7k)(2m - m_1)m_1^3 + 4(k - m)(m^2 + 4km + k^2)m_1^2 \\ &\quad - 2km(m^2 + 6km + 2k^2)m_1 + k^2(m^3 + 2km^2 + 4k^2m + 2k^3). \end{aligned} \quad (9)$$

Since the denominator of (8) is positive and  $k^2(2m_1 - m)$  in the numerator of (8) is positive for  $m_1 > \frac{m}{2}$ , the sign of (8) is equal to that of  $X_5$ .

From (9), we obtain

$$\lim_{m_1 \rightarrow m} X_5(m_1) = k^2(m^3 + 2km^2 + 4k^2m + 2k^3), \quad (10)$$

$$\lim_{m_1 \rightarrow \frac{m}{2}} X_5(m_1) = \frac{(m + 2k)^3(2k^2 + km - 2m^2)}{8}. \quad (11)$$

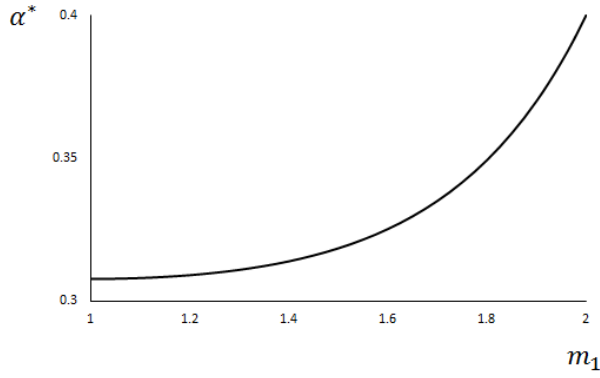
(10) is positive. (11) is nonnegative (negative) if  $m \leq m^*$  ( $m > m^*$ ).

Because (8) is always positive when  $m_1$  is close to  $m$  and negative when  $m_1$  is close to  $m/2$  as long as  $m > m^*$ , Proposition 3(ii) holds.

Suppose that  $m \leq m^*$ . Differentiating  $X_5(m_1)$  with respect to  $m_1$  yields

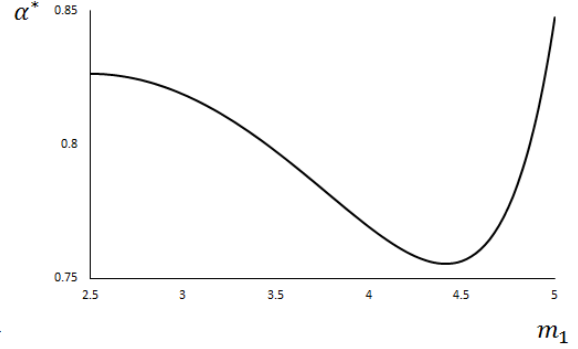
$$\frac{\partial X_5(m_1)}{\partial m_1} = 2(2m_1 - m)\{2m_1(2m + 7k)(m - m_1) + k(m^2 + 6km + 2k^2)\}. \quad (12)$$

This is nonnegative for  $m_1 \in [\frac{m}{2}, m]$  and strictly positive for  $m_1 \in (\frac{m}{2}, m]$ . Therefore,  $X_5(m_1)$  is positive for  $m_1 \in (\frac{m}{2}, m]$  if  $m \leq m^*$ . This leads to Proposition 3(i). ■



Monotone result:  $k=2, m=2$ .

Figure 1



Non-monotone result:  $k=2, m=5$ .

Figure 2

Figure 1 (2) illustrates the case of monotone (nonmonotone) relationship between  $\alpha^*$  and  $m_1$ .

Lin and Matsumura (2012) and Matsumura and Okamura (2015) have already shown that the optimal degree of privatization is increasing with the number of the private firms when firms are homogeneous. A larger  $\alpha$  decreases the output of firm 0 because that firm is less concerned with consumer surplus (Matsumura, 1998). Through strategic interaction, it increases the output of each private firm. As long as the marginal cost is higher in the public firm than in each private firm, this production substitution improves the production efficiency in the industry and thus improves welfare. We call this effect the “production-substitution effect.” Simultaneously, an increase in  $\alpha$  reduces the total output and reduces welfare. We call this effect the “total output effect.” When the number of private firms is larger, the output of each private firm is smaller, and thus, the marginal cost of each private firm is smaller. Therefore, the more private firms, the stronger this welfare-improving production-substitution effect is. When the number of private firms is larger, the total output is larger, and thus, a welfare loss caused by a reduction of total output is smaller. These two effects yield the result that the optimal degree of privatization is increasing with the number of private firms.

Proposition 1 states that an increase in  $m_1$  reduces the total output. Thus, by the total

output effect, the optimal degree of privatization is decreasing in  $m_1$ . Proposition 2 states that an increase in  $m_1$  increases the price–cost margin in the larger firm (firm 1) and reduces it in the smaller firm (firm 2). Therefore, production substitution from firm 0 to firm 1 (firm 2) is more (less) important as  $m_1$  increases. The slope of the reaction curve is more (less) steep as  $m_1$  increases, and an increase in  $\alpha$  more (less) significantly affects firm 1’s (firm 2’s) output. Therefore, the production–substitution effect is more effective as  $m_1$  increases, and this effect dominates the total output effect, especially when  $m_1$  is large. Consequently, the optimal degree of privatization is increasing in  $m_1$ .

## 4 Concluding remarks

In this study, we investigate the relationship between a privatization policy and the asymmetry among private firms that compete with a public firm. We find that the optimal degree of privatization of the public firm is either increasing with or has a nonmonotone (U-shape) relationship with the degree of asymmetry among private firms.

In this study, we propose a reasonable model formulation allowing cost difference among private firms. In the literature on mixed oligopolies, a cost difference between public and private firms is often assumed<sup>8</sup>, most works do not consider a cost difference among private firms. The cost asymmetry among private firms may have a significant implication in mixed oligopolies, and future research needs to investigate this problem in other contexts in mixed oligopolies.

Our triopoly model is the simplest model allowing a cost difference among private firms. However, even this model requires some messy calculations, and extending our analysis to a more general oligopoly model is a challenging task. However, as Haraguchi and Matsumura (2016) showed, the property of mixed oligopoly may change as the number of private firms exceeds a critical value. Therefore, extending our analysis to an  $n$ -firm oligopoly may be a promising future research topic.

In this study, we assume that both firms are domestic. In the literature on mixed oligopolies,

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<sup>8</sup>See Matsumura and Shimizu (2010). For the empirical and theoretical works discussing the cost difference between public and private firms, see Megginson and Netter (2001) and Matsumura and Matsushima (2004).

the nationality of the private firms affects the equilibrium outcomes, especially affecting the optimal privatization policy.<sup>9</sup> Extending our analysis to this direction remains a scope for future research.

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<sup>9</sup>Whether the private firm is domestic or foreign often yields contrasting results in the literature on mixed oligopoly. See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), and Bárcena-Ruiz and Garzón (2005 a,b). The optimal degree of privatization is decreasing with the foreign ownership rate in private firms when the number of private firms is given exogenously (Lin and Matsumura, 2012), while it is increasing in free-entry markets (Cato and Matsumura, 2012).

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